A formal analysis of Correspondence Theory

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Introduction

This paper investigates the computational complexity of Correspondence Theory, as put forth by [1], which explicitly recognizes the correspondence between underlying and surface elements. Assuming Correspondence Theory, we have four distinct results:

- GEN is provably not MSO-definable.
- For a given input w, GEN(w) is provably MSO-definable
- In fact, correspondence structures for a given input w are provably FO-definable with $(<)$.
- Typical underlying representation (UR) to surface representation (SR) maps can be directly described with FO logic without recourse to optimization.

Our approach is similar in spirit to [2], which uses logic to formalize OT constraints. The major difference is that we employ language-specific inviolable constraints, cf. [3].

Correspondence Theory

The Correspondence Theory approach described in [1] characterizes GEN as a function which maps a UR u to a set of ordered triples, as in (1).

(1) $Gen(/\text{abc}/) \rightarrow \{(/\text{abc}/, [\text{abc}], \mathcal{R}_1), (/\text{abc}/, [\text{abcd}], \mathcal{R}_2), (/\text{abc}/, [\text{abcd}], \mathcal{R}_3), \ldots\}$

Within each triple lies another relation ($\mathcal{R}_i$), which maps the segments in the input to those in the output candidates. The fully faithful mapping is common ($/a_1b_2c_3/ \rightarrow a_1b_2c_3$), but there are unboundedly many other possible relations, some examples of which are shown in (2). Segments in the input can have zero, one, or multiple corresponding segments in an output candidate, and these segments may or may not be identical. The triples are sometimes represented as in (2).

(2) $/a_1b_2c_3/ \rightarrow a_1b_2$
$/a_1b_2c_3/ \rightarrow a_1b_2a_0c_3$
$/a_1b_2c_3/ \rightarrow i_1b_2c_3$
$/a_1b_2c_3/ \rightarrow a_1b_2c_3c_0c_0c_0$

The subscript 0 indicates that there is no correspondent in the input.

The GEN function is not MSO-definable

The core reason is stated in [4] as Fact 1.37, where U is an input candidate, and S is the set of output candidates: "For every monadic second-order transduction f there exists an integer k such that, if f transforms a relational structure U into a relational structure S, then $|S| \leq k \times |U|$."
In other words, the size of the set of possible output candidates must be bounded in size. However, it is not.

The candidates are MSO-definable

The *set* of output candidates produced by Gen for a given input *is* MSO-definable. Letting Σ represent the alphabet and N represent the set of indices used for a given input plus the subscript 0 to indicate no correspondent, the set of possible output candidates for a given input is $\Delta = (\Sigma \times \text{N})^*$. This is because the set of logically possibly strings over a finite alphabet is MSO definable.

Correspondence structures are FO-definable

In fact, the set of output candidates created by Gen for a given input (e.g. /kætz/) can be modeled by using a set of FO-definable constraints (3)–(7) over a relational structure, with precedence relations.

(3) Precedence only between nodes on same tier:
$edge_<(x,y) \rightarrow [[u(x) \wedge u(y)] \vee [s(x) \wedge s(y)]]$

(4) Correspondence only between different tiers:
$edge_c(x,y) \rightarrow u(x) \wedge s(y)$

(5) All nodes are either on UR or SR:
$(\forall x)[u(x) \vee s(x) \wedge \neg[u(x) \wedge s(x)]]$

(6) SR nodes form a string:
$(\forall x,y)[[s(x) \wedge s(y)] \rightarrow [edge_<(x,y) \vee edge_<(y,x)]$

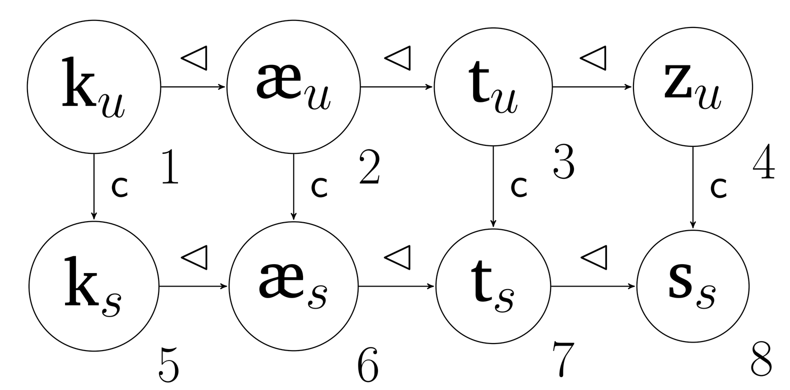
(7) UR is /kætz/:
$(\exists v,x,y,z)[edge_<(v,x) \wedge edge_<(x,y) \wedge edge_<(y,z) \wedge u(v) \wedge k(v) \wedge \text{æ}(\text{x}) \wedge t(y) \wedge z(z)]$

Typical mappings from UR to SR using FO constraints

In order to describe specific phonological alternations, we need language-specific, inviolable constraints on the relational structures described in our previous result.
Below we describe canonical processes of assimilation, insertion, and deletion, using this model. The constraints are formed as statements that ban the crucial substructures,

English Voice Assimilation

(8)



(9) A voiceless sound followed by a voiced obstruent:
$T_xD(x) \overset{\text{def}}{\equiv} (\exists y)[edge_{\triangleleft}(x,y) \wedge voiceless(x) \wedge voiced(y) \wedge obstruent(y)]$

(10) A voiceless sound followed by a voiceless obstruent:
$T_xT(x) \overset{\text{def}}{\equiv} (\exists y)[edge_{\triangleleft}(x,y) \wedge voiceless(x) \wedge voiceless(y)]$

(11) An underlyingly voiceless sound followed by a voiced obstruent has to surface as one followed by a voiceless obstruent:
$(\exists x,y)[(T_xD(x) \wedge u(x) \wedge edge_c(x,y)) \rightarrow T_xT(y)]$

Constraint (11) describes voice assimilation in English with a conditional statement. Alternatively, we can also describe the same process with banned substructures. Below are a few of them:

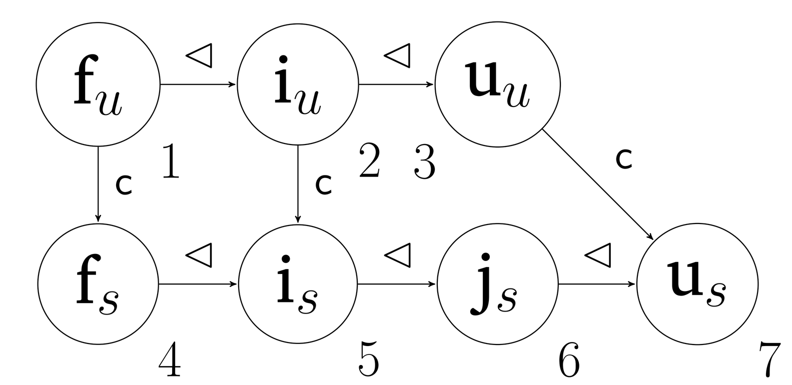
(12) $\neg(\exists x,y)[T_xD(x) \wedge u(x) \wedge edge_c(x,y) \wedge T_xD(y)]$

(13) $\neg(\exists x,y)[T_xD(x) \wedge u(x) \wedge edge_c(x,y) \wedge \neg(\exists z)[edge_{\triangleleft}(y,z)]]$

(14) $\neg(\exists x,y,z)[T_xD(x) \wedge u(x) \wedge edge_c(x,y) \wedge edge_{\triangleleft}(y,z) \wedge \neg(\exists v)[edge_c(v,z)]]$

Hungarian [j]-insertion [5]

(15)

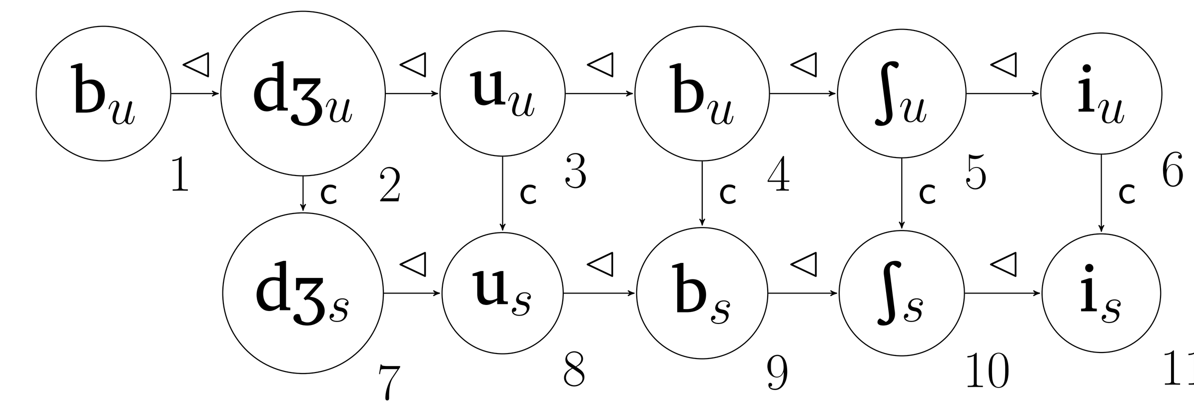


(16) $v_xv(x) \overset{\text{def}}{\equiv} (\exists y)[edge_{\triangleleft}(x,y) \wedge V(x) \wedge V(y)]$

(17) $(\exists x)[v_xv(x) \wedge u(x)] \rightarrow \exists y,z[edge_c(x,y) \wedge edge_{\triangleleft}(y,z) \wedge j(z)]$

Tibetan consonant-deletion [6]

(18)



(19) $c_xc(x) \overset{\text{def}}{\equiv} (\exists y)[edge_{\triangleleft}(x,y) \wedge C(x) \wedge C(y) \wedge \neg(\exists z)[edge_{\triangleleft}(z,x)]]$

(20) $(\exists x)[c_xc(x) \wedge u(x)] \rightarrow \neg(\exists y)[edge_c(x,y)]$

References

[1] John McCarthy and Alan Prince. *Faithfulness and reduplicative identity.* 1995. [2] Christopher Potts and Geoffrey K Pullum. "Model theory and the content of OT constraints." *Phonology,* 19(03):361-393, 2002. [3] Adam Jardine. "Locality and non-linear representations in tonal phonology". PhD thesis, University of Delaware, 2016. [4] Bruno Courcelle and Joost Engelfriet. "Graph structure and monadic second-order logic: a language-theoretic approach", volume 138. Cambridge University Press, 2012. [5] Peter Siptar. "The phonology of Hungarian vowel clusters". *Magyar Nyelv (Hungarian Language),* 3(101):282-304, 2005. [6] Morris Halle and George Clements. *Problem book in phonology: a workbook for introductory courses in linguistics and in modern phonology.* MIT Press, 1983.