A formal analysis of Correspondence Theory

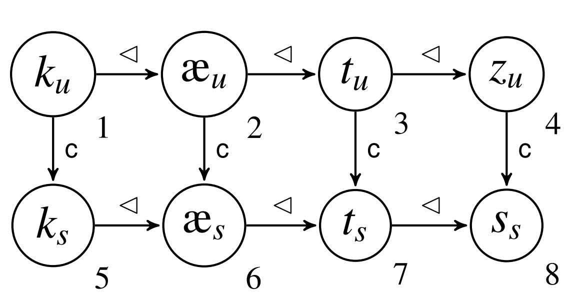
The goal of this paper is to investigate the computational complexity of Correspondence Theory, as put forth by McCarthy and Prince (1995), which explicitly recognizes the correspondence between underlying and surface elements. We have three distinct results. First, we show that the GEN function, assuming Correspondence Theory, is not definable using Monadic Second Order logic (MSO). On the other hand, we show that the *set* of output candidates for a given input is in fact definable in First-Order (FO) logic, given a relational representation. Lastly, we find that typical underlying representation (UR) to surface representation (SR) mappings can be directly described with FO logic without recourse to optimization. Our approach is similar in spirit to Potts and Pullum (2002), which uses logic to formalize OT constraints. The major difference is that we employ language-specific inviolable constraints, cf. Jardine (2016).

Correspondence Theory is widely used in OT analyses (Kager, 1999). It relies on GEN, which maps input forms to candidates, considered as input-output pairs (Prince and Smolensky, 2004). Earlier computational analyses of GEN (Frank and Satta, 1998; Riggle, 2004) understood it as a regular relation. MSO-definable functions are not the same as regular relations (Engelfriet and Hoogeboom, 2001), therefore this study supplements previous research.

One way of defining complexity is through a logical hierarchy. For instance, MSO logic is more complex than FO logic, which is more complex than propositional logic (Rogers and Pullum, 2011). MSO-definabilty has been argued to be an important quality for phonological patterns (Heinz, forthcoming). In any MSO definable function, the size of the output must be bounded by some positive integer value times the size of the input (Engelfriet and Hoogeboom, 2001). Since GEN's output is not bounded, it cannot be an MSO-definable function.

However, the *set* of candidates produced by GEN for a specific input w is in fact FO-definable, if we represent the UR-SR correspondence in a relational structure, with constraints (1b–1d). The underlying form and the surface form are represented in two tiers in a graph, and each node is labeled for the sound it represents and also for being part of the UR (u) or the SR (s). Constraint (1b) specifies that edges labeled $\triangleleft$ can only connect nodes in the same tier. Constraint (1c) requires that edges labeled c force the correspondence to be between the underlying and surface tiers. Constraint (1d) spells out the specific input w, which is /kætz/ in this example. The correct mapping is shown in (1a), which is one of the infinitely many possible candidates for /kætz/. If we consider the set of all possible structures which do not violate constraints (1b–1d), then this is exactly the candidate set produced by GEN for the input /kætz/. Note that (1b-1d) are FO definable.

(1) a.



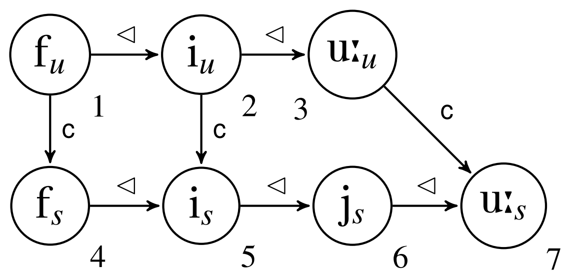
b. $edge_{\triangleleft}(x,y) \rightarrow [[u(x) \wedge u(y)] \vee [s(x) \wedge s(y)]]$

c. $edge_c(x,y) \rightarrow u(x) \wedge s(y)$

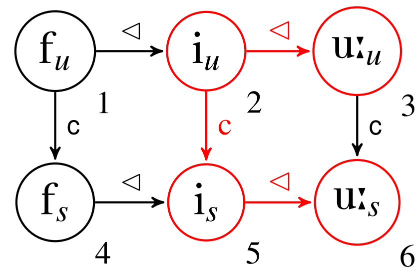
d. $(\exists v,x,y,z)[edge_{\triangleleft}(v,x) \wedge edge_{\triangleleft}(x,y) \wedge edge_{\triangleleft}(y,z) \wedge u(v) \wedge u(x) \wedge u(y) \wedge u(z) \wedge k(v) \wedge æ(\mathrm{x}) \wedge t(y) \wedge z(z)]$

Our third result shows that if we eliminate Constraint (1d) we can model all potential correspondences between URs and SRs directly, provided we add language-specific inviolable constraints on these correspondence structures. This model can describe a variety of canonical phonological processes, including Tibetan consonant deletion (Halle and Clements, 1983), nasal assimilation (Onn, 1980), and Hungarian [j] epenthesis in vowel hiatus (Siptar, 2005), (shown in (2)). To get the correct SR [fijuː] as in (2a), we need to ban graphs like (2b) where underlying vowel hiatus is not fixed, as well as those that use any repair strategy which does not insert [j]. (2c) defines structures where a vowel is followed by another vowel. We use this definition to write our constraint in (2d), which is violated in (2b), but not in (2a). Other constraints, not shown here, rule out all other correspondence structures which have /fiuː/ underlyingly.

(2) a.



b.



c. $\mathrm{v}_x\mathrm{v}(x) \overset{\text{def}}{\equiv} (\exists y)[edge_{\lhd}(x,y) \wedge vowel(x) \wedge vowel(y)]$

d. $\neg(\exists x,y)[v_x v(x) \wedge v_x v(y) \wedge edge_c(x,y)]$

In this paper we have shown that the formal properties of Correspondence Theory depend on the particulars. The complexity of the GEN function itself is not MSO-definable, while each output candidate set is. However, we also show that GEN is not required to encode individual UR-SR mappings, as these can be defined with various FO constraints. As we will discuss, many of these resemble standard OT faithfulness constraints. Using Correspondence Theory this way can account for a wide variety of phonological processes, without resorting to optimization. Whether these mappings can be defined in a more restrictive logic is a subject for future research.

References • Engelfriet, J. and H. J. Hoogeboom (2001). MSO-definable string transductions and two-way finite-state transducers. *ACM Trans. Comput. Logic* *2*, 216–254. • Frank, R. and G. Satta (1998). Optimality theory and the generative complexity of constraint violability. *Computational Linguistics* *24*(2), 307–315. • Halle, M. and G. Clements (1983). *Problem book in phonology: a workbook for introductory courses in linguistics and in modern phonology*. MIT Press. • Heinz, J. (forthcoming). The computational nature of phonological generalizations. In L. Hyman and F. Plank (Eds.), *Phonological Typology*, Phonetics and Phonology. Mouton. • Jardine, A. (2016). *Locality and non-linear representations in tonal phonology*. Ph. D. thesis, University of Delaware. • Kager, R. (1999). *Optimality Theory*. MIT Press. • McCarthy, J. J. and A. Prince (1995). Faithfulness and reduplicative identity. • Onn, F. M. (1980). *Aspects of Malay phonology and morphology: A generative approach*. Universiti Kebangsaan Malaysia. • Potts, C. and G. K. Pullum (2002). Model theory and the content of ot constraints. *Phonology* *19*(03), 361–393. • Prince, A. and P. Smolensky (2004). *Optimality Theory: Constraint interaction in generative grammar*. Blackwell Publishing. • Riggle, J. A. (2004). *Generation, recognition, and learning in finite state Optimality Theory*. Ph. D. thesis, UCLA. • Rogers, J. and G. K. Pullum (2011). Aural pattern recognition experiments and the subregular hierarchy. *Journal of Logic, Language and Information* *20*(3), 329–342. • Siptar, P. (2005). The phonology of Hungarian vowel clusters. *Magyar Nyelv (Hungarian Language)* *3*(101), 282–304.